CONTINUED FRACTIONS AND OPTIMIZATION OF HUYGNES' PLANETARIUM MODEL

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Mercury
ΔH = -1359,17°
ΔP = 538,06°

Venus
ΔH = -1443,56°
ΔP = 1304,53 °

Earth is a refference

The aim of this project is to get the planetarium model which more corresponds to reality by using continued and intermediate fraction approximations.

Mars
ΔH = -66,44°
ΔP = 61,93 °

Jupiter
ΔH = -7,38°
ΔP = 7,11 °

Saturn
ΔH = -251,62°
ΔP = 80,44 °

1. Introduction

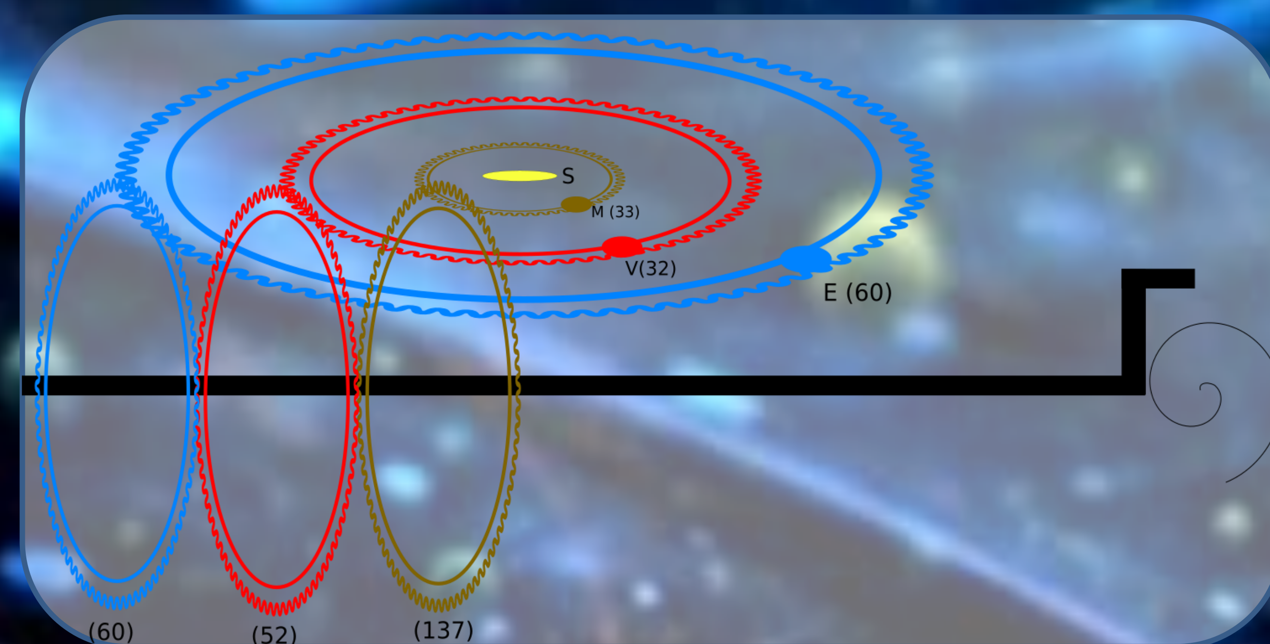
The Huygens' planetarium is a model of orbiting planets based on a system of gears which is developed in 1680. It is in a form of octagon which consists of six planets known by then. In order to get the right number of gear teeth Huygens' used ratio of tropical orbit periods, but by that way he got large numbers and it was impossible to make gears with that numbers of gear teeth. With the aim of solving that problem he decided to replace these numbers with a smaller ones. And he did it with a help of *continued fractions*.

Those fractions are expressions of this form :

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_n}}}}$$

where $a_0 \in Z \wedge a_i \in N \ (i \geq 1)$.

Mechanism of gears for first three planets after replacing large numbers with a continued fractions is shown on this picture ⟶



For continued fractions, *intermediate fractions* are introduced:

$$\alpha' = \frac{p_n'}{q_n} = [a_0; a_1, a_2, \dots, a_{n-1}, a_n']$$

where $a_n' \in N \wedge 0 < a_n' < a_n$. These fractions were unknown at the period in which Huygens lived. German mathematician Oskar Perron discovered them and the theory was developed by Russian mathematician Alexandr Kinchin.

Formulas for calculating numerators and denominators of continued fractions:

$$p_n = a_n \cdot p_{n-1} + p_{n-2}$$

$$q_n = a_n \cdot q_{n-1} + q_{n-2}$$

p_n - numerator of continued fraction
q_n – denominator of continued fraction
a_n - continued fraction digit

Properties of continued fractions:

Property 1. $(p_n, q_n) = 1, (n \in N_0) \wedge q_n \nearrow^{\infty} (n \longrightarrow \infty)$

Property 2. $\lim_{n\to\infty}\left|\frac{p_{n+1}}{q_{n+1}} - \frac{p_n}{q_n}\right| = \lim_{n\to\infty}\frac{1}{q_n \cdot q_{n+1}} = 0$

Property 3. $\frac{p_0}{q_0} < \frac{p_2}{q_2} < \cdots \leq \alpha \leq \cdots < \frac{p_3}{q_3} < \frac{p_1}{q_1}$

Property 4. $\left|\alpha - \frac{p_n}{q_n}\right| < \frac{1}{q_{n-1}^2}$

Property 5. Every best approximation of the first kind is **continued fraction** or **intermediate fraction**

2. Methodology

In order to faster finding continued and intermediate fraction approximations which are used for improvement of Huygens' planetarium model an algorithm for finding them is created and implemented in Pyhton.

3. Results and discussion

Data for tropical orbit period given by NASA are used for making ratio for each planet and than these numbers are put in program which found all approximations for these numbers. From these approximations Huygens', first larger and smaller approximations are allocated. These approximations are recognized as continued or intermediate fraction approximations. In order to get the more precise planetariume model first larger approximation is chosen.

4. Conclusion

In this paper algorithm which finds continued fraction approximations and intermediate fraction approximations was created. It was successfully executed to find the best possible optimizations for Huygens' planetarium model. Obtained results show that by choosing larger alternatives it is able to make calculations for the model of planetarium which more corresponds to reality.

5. References

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